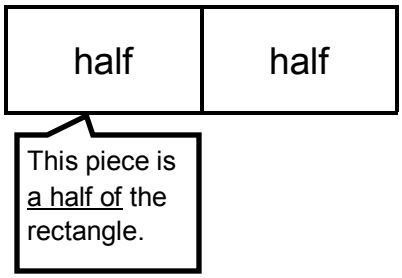
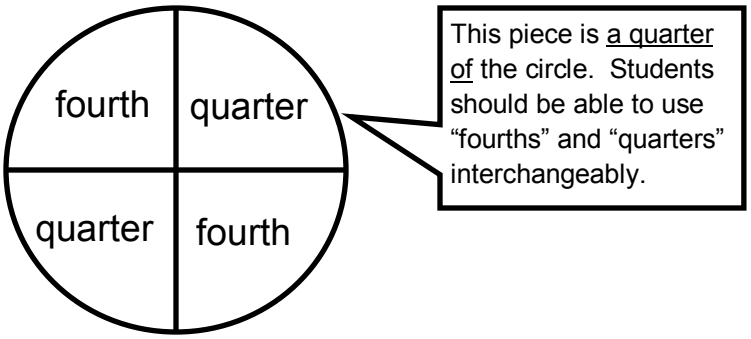
Fractions Progression

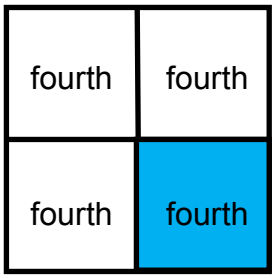
1st Grade: Students begin their understanding of fractions in this grade level by partitioning (folding, drawing lines, or cutting) circles and rectangles to two or four **equal** shares. Students in this grade level are not expected to record fractions in standard notation (½ or ¼...Research shows that students develop a better understanding if they are not exposed to the standard notation at first. Please wait!), but students are expected to refer to the pieces as halves, half of, fourths, fourth of, quarters (also for fourths), or quarter of the whole circle or rectangle. Students at this grade level should also understand that a half of a circle or rectangle is bigger than a fourth of the same circle or rectangle.



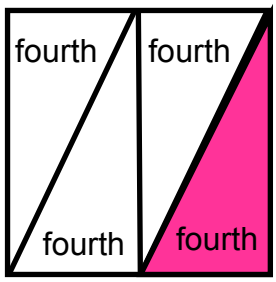


2nd Grade: Students in this grade level continue their work with fractions by partitioning (folding, drawing lines, or cutting) circles and rectangles to two, three, or four **equal** shares. Students in this grade level are not expected to record fractions in standard notation (½ or ¼...Research shows that students develop a better understanding if they are not exposed to the standard notation at first. Please wait!), but students are expected to refer to the pieces as halves, half of, thirds, third of, fourths, fourth of, quarters, or quarter of the whole circle or rectangle. Students at this grade level should also understand that when the same rectangle or circle is shared with more people, each person gets a smaller piece, so a third is smaller than a half, and a fourth is smaller than a third. Also at this grade level, students explore the idea that fractional parts can look differently, but still represent the same amount. Students need to cut different "fourths" and "halves" to see that they are the same amount even the whole is partitioned differently (see below). Students at this grade level also begin to relate the idea that if I partition a shape into halves by folding, each part is one half. If I fold the halves in half, now I have four equal shares or quarters. This allows students to begin to understand the relationship between halves and fourths.

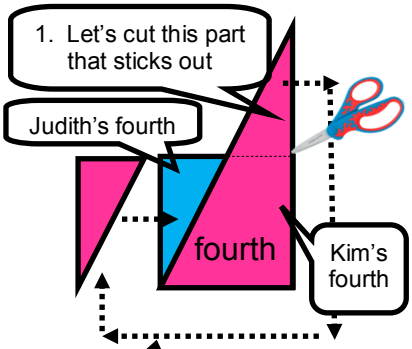
Judith's Fourths:



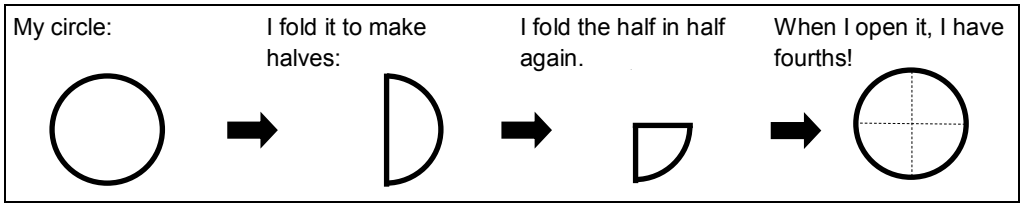
Kim's Fourths:



How can both of these be fourths? It looks like both were partitioned into equal parts, but the parts just don't look the same. Let's cut one of the triangle fourths and put it over the square fourth to see if it's the same.



2. Notice that if we move the piece we cut from Kim's fourth, it will lie exactly over Judith's fourth. So they are the same

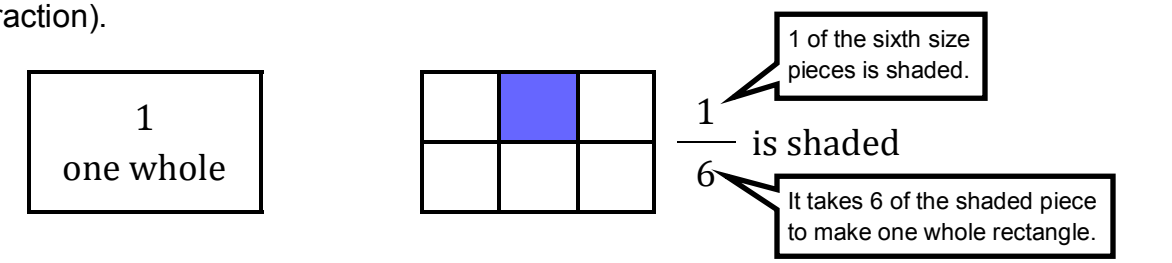


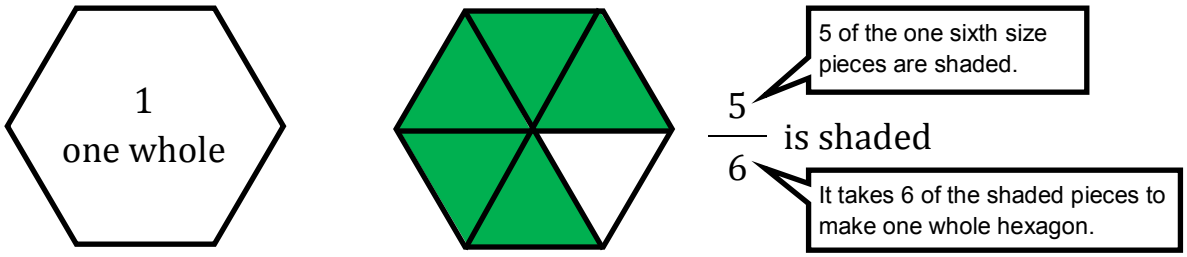
3rd Grade:

Student work with fractions intensifies in this grade level. Student continue to work with area models (circles, rectangles, and other two-dimensional shapes) and add work with fractions on number lines. Students in this grade level only need to work with halves, fourths, eighths, thirds, and sixths.

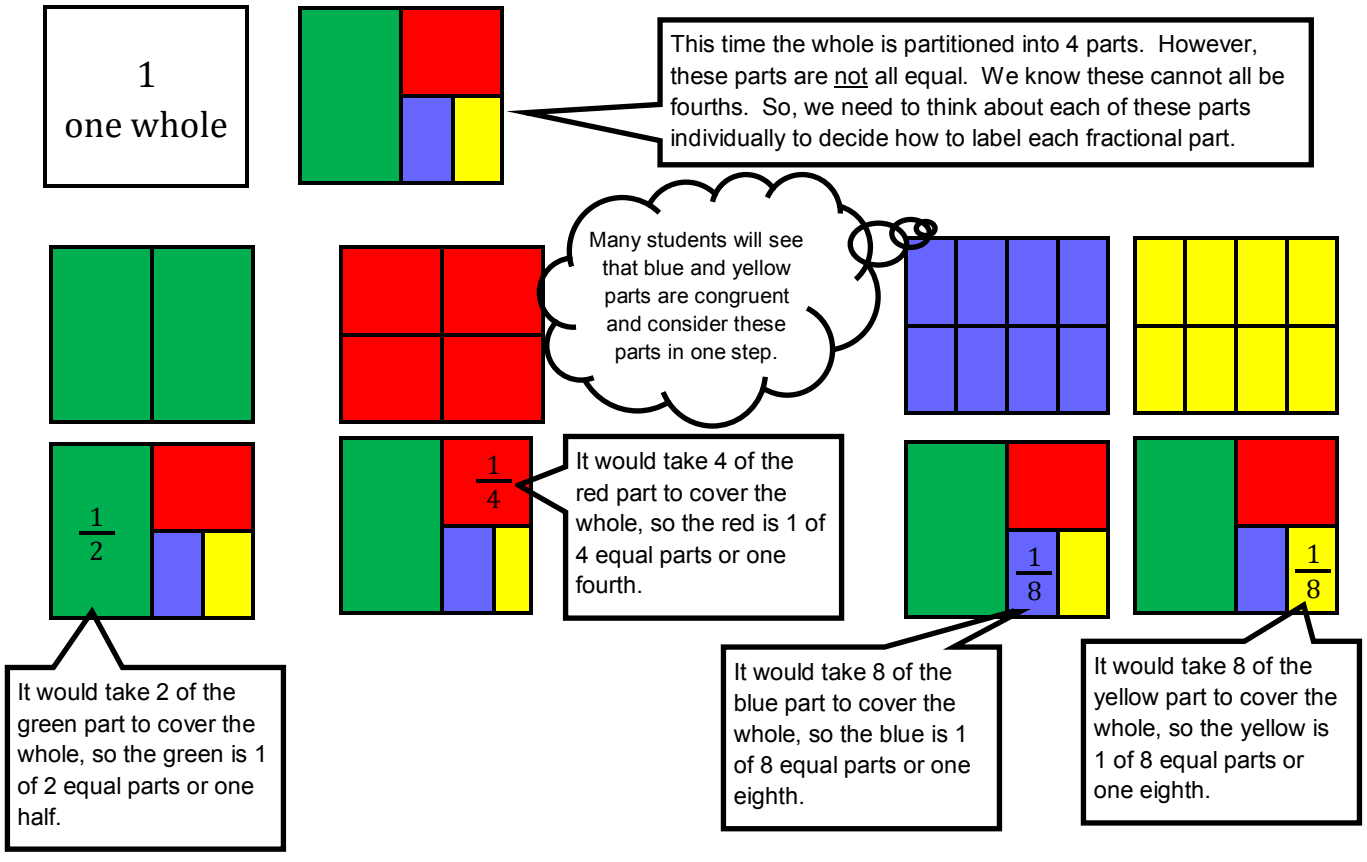
Several very important fraction ideas are key for third grade students:

- A fraction is represented in standard form by recording the number of parts that size that it takes to make a whole as the **denominator** (bottom number of a fraction), and by recording the number of parts the size of the denominator being referred to as the **numerator** (top number of the fraction).



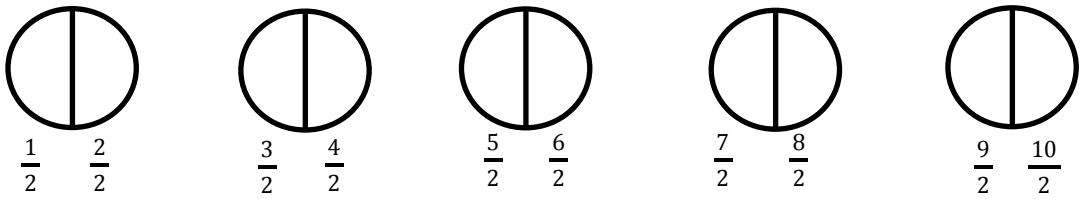


- One whole can be composed using parts that are not equal. We can name the parts if we know how many of that size it takes to make a whole.



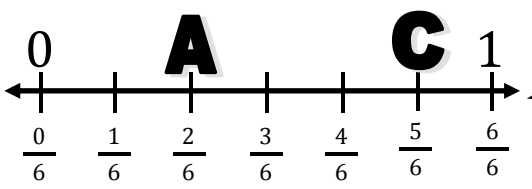
3rd Grade (continued):

- Fractions are numbers, so we can count fractions.



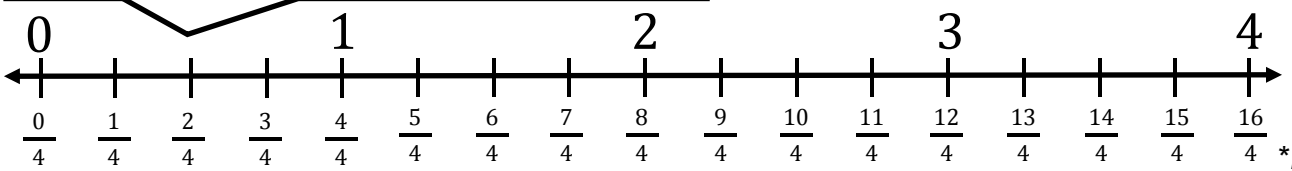
If one circle is 1 whole, how many halves above? 1 half, 2 halves, 3 halves, 4 halves, 5 halves, 6 halves, 7 halves, 8 halves, 9 halves, 10 halves.

- Fractions are numbers, so we can locate a fraction on number line just like we can locate whole numbers such as 0, 1, 2, 3, etc. Fractions are located between whole numbers on a number line.



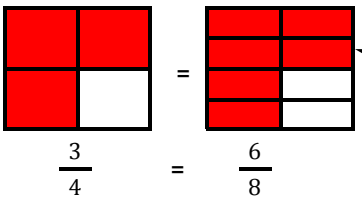
This number line is partitioned into 6 equal parts between 0 and 1. So the distance from one tick mark to the next is one sixth. Students need to count the spaces between tick marks, not the tick marks themselves. The letter C is located at five sixths. The distance from A to C is three sixths, since a hop from one tick mark to the next is one sixth.

This number line is partitioned into 4 equal parts between 0 and 1, so this number line is counting by fourths.



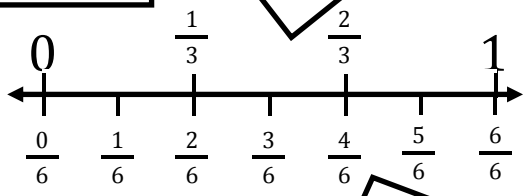
*Note *that third grade students do not have to change improper fractions to mixed numbers, but they do need to recognize when fractions are equivalent to one whole (see more below).*

- Sometimes fractions with different numerators and denominators represent the same amount and are located at the same place on a number line. We call those **equivalent** fractions. Note that third grade students should always use visual models such as partitioned rectangles, circles, or number lines to show why two fractions are equivalent.



These two fractions are partitioned differently, but have the same amount shaded. They are **equivalent** fractions.

The top of this number line is partitioned into 3 equal parts. So it is counting thirds.



Third grade students also need to recognize fractions equivalent to whole numbers. On the number line to the right, students need to recognize that six sixths is **equivalent** to one whole. In the diagram at the top of the page students can see that ten halves is **equivalent** to 5 whole circles.

The bottom of this number line is partitioned into 6 equal parts. So it is counting sixths. Since one third and two sixths are located at the same spot, they are **equivalent** fractions.

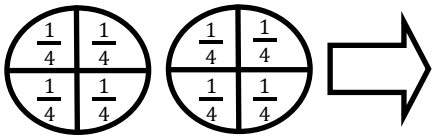
Why is $\frac{3}{1}$ the same as 3?

$\frac{3}{1} = 3$

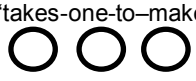
The denominator tells how many equal parts needed to make a whole. For example,

$\frac{8}{4} = 2$ It takes 4 equal parts to make one whole circle in the shapes below.

So 8 'takes-four-to-make-a-whole' size pieces will make 2 wholes.



So, if we have $\frac{3}{1}$ that means that each whole is made with 1 piece (not 2 like halves or 3 like thirds). Essentially, each piece is a whole. We read this fraction as "3 wholes," because 3 'takes-one-to–make-a-whole' size pieces is 3 wholes.



3rd Grade (continued)

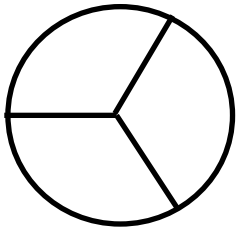
- Fractions can be compared to see which is greater.
  1. A fraction is greater if we have more of the same size piece: $\frac{5}{6}$ is greater than $\frac{2}{6}$.

$\frac{5}{6} > \frac{2}{6}$

If the denominator (the size of the pieces) is the same, then the more you have (numerator), the greater the area that is shaded.

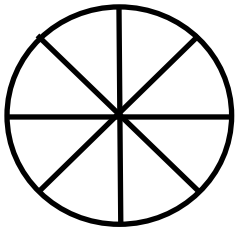
  2. A fraction is greater if it represents the same whole shared by fewer people. $\frac{1}{3}$ is the size part only shared by 3 people, so it is greater than $\frac{1}{8}$ because it is a part of the same whole shared by 8 people. 3 people sharing get bigger pieces than 8 people sharing!

3 people share a pizza



$\frac{1}{3} > \frac{1}{8}$

8 people share a pizza



If the numerator is the same, but the denominator is different, then the one that has a smaller denominator is actually bigger. This is because fewer people are sharing the equal parts.

Each person eats $\frac{1}{3}$ of the pizza.

Each person eats $\frac{1}{8}$ of the pizza.

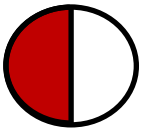
Note: This thinking can be extended to same numerators greater than 1.

If I have 2 one third size pieces $\frac{2}{3}$, I know it's bigger than 2 one eighth size pieces $\frac{2}{8}$ because each of the one third pieces are bigger than each of the one eighth pieces. Use the diagram above. Two of the thirds will be bigger than two of the eighths.

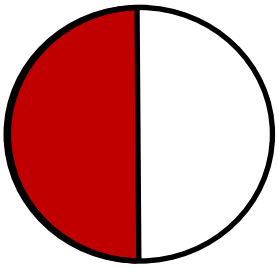
$$\frac{2}{3} > \frac{2}{8}$$

- Fractions can only be compared if they represent a fraction of the same-size whole.

Tori ordered a small pizza.
She ate half of the pizza.



Rylie ordered a large pizza.
She ate half of the pizza.



Both girls ate half of their pizza, but half is not the same in these two cases because Tori's whole is smaller than Rylie's whole. Any time we compare two fractions, we must make sure we are comparing the same size whole.